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Disperse Phase Motion in Neutrally Buoyant and Zero-Gravity Pipe Flows

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Introduction

IT has been shown by Sridhar et al.[1] that the dynamics of steady, fully developed, turbulent, dispersed liquid-vapor duct flows in zero gravity can be simulated by using two immiscible, neutrally buoyant liquids in an Earth-based flow facility. Such simulation experiments have been used to predict the frictional pressure drop in a zero-gravity environment. Theoretical analysis showed that the mean discrete-phase velocity and the local mean continuous-phase velocity are identical in liquid-vapor flow in zero gravity and in liquid-liquid flows of neutral buoyancy on Earth. This finding forms the cornerstone of the simulation scheme. However, only a limited amount of experimental data supporting this finding was reported.[1]

The equation of motion for a Stokesian sphere in unsteady, nonuniform flow was rigorously derived by Maxey and Riley.[2] The equation for the dispersed phase used in the analysis of Sridhar et al.[1] was based on the equation due to Maxey and Riley and modified to account for a multiparticle system and non-Stokesian motion. The modification was not rigorous. The equation for the continuous phase was formulated by subtracting the dispersed phase equation from the momentum equation for the mixture. The results show that the mean continuous phase velocity U_c is equal to the mean discrete-phase velocity U_d, for both 0-*g* liquid-vapor flows and 1-*g* neutrally buoyant flows. Because of the fundamental nature of this result, and its importance to microgravity simulation, additional experiments were conducted for its validation.

Related Work

Batchelor et al.[3] made extensive measurements of the mean velocity of single, neutrally buoyant particles in fully developed turbulent pipe flow of water and related the mean velocity to the discharge velocity. They were interested in comparing the Lagrangian average of the velocity of a material element of the fluid with the Eulerian average of the fluid velocity at fixed points. The particle velocity was measured using photographic detectors and timing slits provided along the length of the pipe. The discharge velocity was determined with a measuring tank.

To relate the velocity of a particle to that of the fluid, Batchelor et al.[3] assumed that the mean particle velocity is equal to the mean velocity of the undisturbed fluid at the same location as the center of the particle, provided that the particle is so small that the mean fluid velocity does not vary appreciably over a distance equal to the particle diameter. For larger particles, they took the mean particle velocity as equal to the fluid velocity averaged over the cross section of the particle. Furthermore, it was recognized that the material element in turbulent pipe flow wandered freely over any region of a cross section and a particle of finite size would not have the same freedom. If the pipe radius is r_0 and the particle is a sphere of radius αr_0, the center of the particle is free to move down the pipe only within a cylinder of radius $(1-\alpha)r_0$. Thus, the probability of finding the center of the particle inside the cylinder is the same for all positions and is zero outside the cylinder. The results of their experiments confirmed the theoretical prediction that the local discharge velocity, the ensemble average of the velocity of a material element, and the velocity of a material element averaged over a long time are all equal. The Lagrangian average of the velocity of a material element was determined by spheres of various sizes and extrapolated to zero size. Thus, a corollary of the investigation by Batchelor et al. is that the mean velocity of the neutrally buoyant spheres in turbulent pipe flow is identical to the mean velocity of the undisturbed fluid at the same location. The latter is to be determined by averaging over the cross section of the sphere. The experiments of Batchelor et al. were for single particles. The purpose of this Note is to demonstrate that the conclusion $U_c = U_d$ remains valid for multiparticle systems.

Experimental Facility

The two neutrally buoyant immiscible liquids used in the experiments were water for the continuous phase and n-butyl benzoate ($C_6H_5COOC_4H_9$) for the dispersed phase (droplets). Two series of experiments were conducted, one with a square test section and another with a circular test section. The square test section had an inside dimension of 50.8 mm on a side and a length of 1.83 m. The circular test section was made by connecting together four 15.4-mm i.d. Pyrex glass tubes, each 1.22 m long. A globe valve located downstream of a centrifugal pump was used to control the flow rate of water through the test section, and monitoring was done using a venturi and a Validyne differential pressure transducer. The velocity of the droplets was measured using particle image velocimetry. The underlying principle behind this technique is to mark time and record the spatial displacements of the droplets. This was achieved by means of an EG&G Type 501 stroboscope and a 35-mm still camera. Details of the setup, instrumentation, and experimental procedure can be found in Sridhar et al.[1]

Results and Discussion

In the first series of experiments, water was pumped through the 50.8-mm square test section at a rate of 1.274×10^{-3} m^3s^{-1} and benzoate was injected into the flow at a rate of 8.83×10^{-6} m^3s^{-1}, giving rise to a range of drop sizes. The area-averaged mixture velocity was 0.497 ms^{-1}, and the flow Reynolds number was 25,200. The stroboscope was flashed at a frequency of 50 Hz, and the shutter of the still camera was open for a duration of 0.4 s. Only droplets in the central one-third of the square cross section, where the water velocity profile was relatively flat, were used. The photographs revealed that the droplets were spherical and almost completely free from distortion. Data were gathered for a total of 48 droplets in the fully developed region with diameters ranging from less than 2 to 5 mm. The results of the average velocities for the four size groups and their standard deviations have already been reported.[1] It was shown that the drop velocities are independent of their sizes, and their variation among the groups is within experimental error. It was also shown that the average water

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velocity in the central one-third of the 50.8-mm square duct is equal to the average drop velocities within experimental error.

A second series of experiments was performed in circular test tubes for which the power-law profiles are known to be valid. Two sets of experiments were conducted in the second series. The first was at a water flow rate of 1.738×10^{-4} m^3s^{-1} and a benzoate flow rate of 1.67×10^{-6} m^3s^{-1}, giving an area-averaged velocity of 0.942 ms^{-1} based on the combined flow rate. The corresponding flow Reynolds number was 13,080. The second set was for a water flow rate of 2.415×10^{-4} m^3s^{-1} with the same benzoate flow rate. The corresponding area-averaged velocity was 1.305 ms^{-1} and the Reynolds number was 18,260.

In both sets, only droplets in the central 36% of the pipe cross section were used. In the first set, 52 drops of diameters less than 1 mm, 63 drops with diameters in the range of 1–2 mm, and 56 drops of diameters larger than 2 mm, a total of 171 droplets, were obtained. The spatial location of the droplets in the axial and radial directions and their size were recorded from the photographs for each exposure. Five measurements were obtained for each droplet, and the average velocity and standard deviation were calculated. The local water velocity in the circular pipe was computed using the seventh power law. Thus, at any radial position r, it is given by $U_c(r) = (\bar{U}_c/0.817)\ (1 - r/r_0)^{1/7}$, where $\bar{U}_c$ is the mean area-averaged water velocity based on the combined flow rate and r_0 is the inner radius of the duct.

The overall accuracy of the droplet velocity measurements depends on the resolution of the photographic record and the repeatability of the measurement technique. The resolution of the object was nearly 38 μm, and the repeatability was within 100 μm. Based on these considerations, the error in the droplet velocity measurement was estimated to be less than 1%. The discharge velocity of water (area-averaged velocity) was determined by monitoring the pressure drop across a calibrated venturi using a precision differential pressure transducer. Its overall measurement accuracy varied from 0.3 to 0.9%.

The droplet velocities at various radial locations are plotted in Fig. 1, along with the velocity profile of water shown by the solid curve. To compare the difference between U_c and U_d for the three size groups, we evaluate the ratio $\Gamma \equiv |U_c - U_d|/U_{max}$, where U_{max} is the centerline velocity of water. For droplets with diameters less than 1 mm, the mean and standard deviation for Γ are 0.0019 and 0.022, respectively. The corresponding values are 0.000005 and 0.028 for droplets in the size range of 1–2 mm and 0.0077 and 0.03 for the droplets greater than 2 mm. Hence, one may conclude that for all practical purposes, $U_c = U_d$. For the population of droplets from which these random samples were taken, students' t-tests were also performed. It was found that the simple result $U_c = U_d$ could be accepted at the 95% confidence level for all size groups.

In the second set of experiments, a total of 138 droplets were tracked at a flow Reynolds number of 18,260. Eighty-

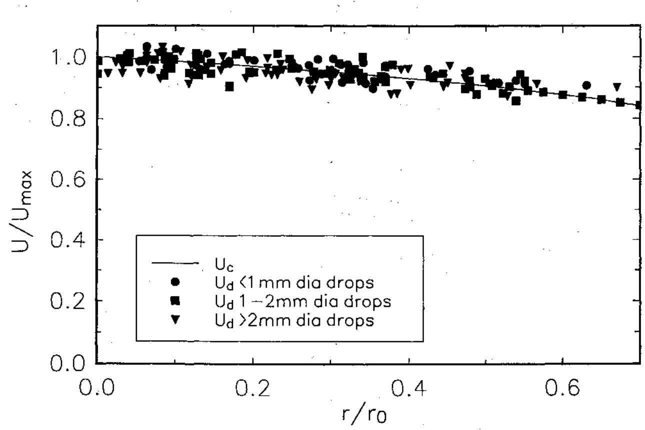

Fig. 1 Velocities of benzoate drops in water in a 15.4-mm i.d. pipe at *Re* = 13,080.

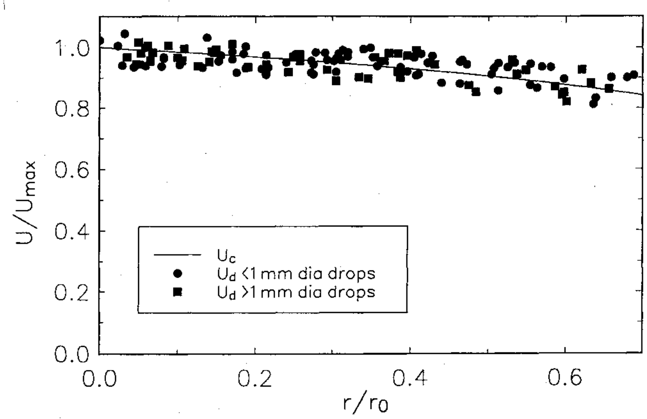

Fig. 2 Velocities of benzoate drops in water in a 15.4-mm i.d. pipe at *Re* = 18,260.

seven of the droplets had diameters less than 1 mm and 51 were larger than 1 mm. The measured droplet velocities and the calculated water velocity profiles are plotted in Fig. 2 for the two groups. For droplets with diameters less than 1 mm, the mean and standard deviation for Γ are 0.0061 and 0.034, respectively. The corresponding values for droplets with diameters greater than 1 mm are 0.0046 and 0.033. Again, the students' t-tests showed that the central result of this investigation, $U_c = U_d$, can be accepted at the 95% confidence level for the population from which the random samples were taken.

The theoretical basis for the simulation scheme for the dispersed flows described by Sridhar et al.[1] is based on one-dimensional conservation equations. Transverse motion of the discrete phase, benzoate droplets in the present case and vapor bubbles in the zero-gravity situation, is ignored all together. Should the transverse motion occur either within or across the wall shear layer, momentum transfer would have taken place between the discrete and the continuous phases. For the water-benzoate system, experiments were conducted to assess the effect of such transverse motion by deliberately varying the diameter of the droplets from less than 1 mm to as large as 30 mm in water flowing in a 45-mm square duct at a Reynolds number of 13,330, while keeping all other flow variables unchanged. The frictional pressure drop in the fully developed region of the test section was measured and the characteristics of the wall turbulence as reflected by the power spectra of the flow-induced wall pressure fluctuations were examined. The very large changes in the droplet sizes had no measurable effect on the frictional pressure drop of the mixture flow (see Table 5 of Sridhar et al.[1]). The effect on the power spectrum was generally small but discernible. In the neutrally buoyant benzoate-water system, the net body force on the droplet is zero. The benzoate droplets generally stay away from the shear layer in the wall region and remain in the core, due to shear lift and droplet rotation in the boundary layer. Visual observation confirmed the absence of droplets in the wall layer. These results suggest that no significant radial motion of the droplet occurs within or across the boundary layer and, hence, the transfer of momentum from this source is indeed negligible.

The presence of the neutrally buoyant benzoate droplets must necessarily have an effect, albeit small, on the turbulence structure of the continuous phase, water in this case. For droplets of diameter d_p, their presence would selectively suppress the motion of eddies of sizes smaller than d_p and fluctuations of high characteristic frequencies. The discernible change in the power spectra of the flow-induced wall pressure fluctuations reported by Sridhar et al.[1] may very well be a revelation of the phenomenon. However, their overall net effect on the frictional pressure drop in fully developed flow is negligible, due to the absence of the droplets in the wall layer.

The foregoing findings are expected to hold for fully developed, bubbly flow in zero gravity. The fact that the benzoate droplets are almost perfect spheres suggests that interfacial tension plays no role in the simulation. The additional fact that $U_c = U_d$ suggests that the ratio of viscosity of the dispersed and continuous phases is also not of importance.

Acknowledgments

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Errata

Eigensolutions Sensitivity for Nonsymmetric Matrices with Repeated Eigenvalues

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DURING production of this paper, some mistakes were not corrected. We regret these errors.

Pages 1326, 1327

In Eqs. (A6), (A7), A9), and (A11), index j runs in the interval $[1, r]$.

In Eq. (A7b), λ_1^{dr} should be multiplied by α_{r0}.